International Evidence On The Oil Price-Real Output Relationship:

Does Persistence Matter?[*]

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Abstract: The literature on the relationship between real output growth and the growth rate in the price of oil, including an allowance for asymmetry in the impact of oil prices on output, continues to evolve. Here we show that a new technique, which allows us to control for both this asymmetry and also for the persistence of oil price changes, yields results implying that such control is necessary for a statistically adequate specification of the relationship. The new technique also yields an estimated model for the relationship which is more economically interpretable. In particular, using quarterly data from 1976 – 2007 on each of six countries which are essentially net oil importers, we find that changes in the growth rate of oil prices which persist for more than four years have a large and statistically significant impact on future output growth, whereas less persistent changes (lasting more than one year but less than four years) have no significant impact on output growth. In contrast, 'temporary' fluctuations in the oil price growth rate – persisting for only a year or less – again have a large and statistically significant impact on output growth for most of these countries. The results for the single major net oil producer in our sample (Norway) are distinct in an interesting way.

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1. The Oil Price-Output Relationship

Empirical support is mixed for the impact of the price of oil on US macroeconomic variables. Since the 1980s, there have been numerous studies on this relationship, but results are sensitive to both the sample period and the model specification. Mixed results aside, one conclusion is generally accepted: allowing for asymmetry in the relationship is important, and increases in oil price are more important than decreases.

We contribute to this large empirical literature by considering the *persistence* of the changes in oil price. We argue that the growth rate in output responds differently to a temporary change in the growth rate of oil than to a relatively more persistent one. Nordhaus (2007) points out two major channels by which the oil price can affect output. First, an increase in oil price induces inflation; if the central bank tightens monetary policy as a response, then output drops as a result. Second, an increase in oil price can impact consumers as a tax increase. Both of these mechanisms are arguably stronger if the oil price change is permanent rather than transitory.

As is fairly standard in this literature, the basic specification considered in this paper is:

$$\Delta y_{t+1} = \alpha + \gamma \Delta y_t + \beta \Delta o_t + u_t \tag{1}$$

where Δy_{t+1} is the real GDP growth rate from the current quarter to the next and Δo_t is the change in logarithm of the nominal oil price from the previous quarter to the current one; β is often allowed to differ for positive versus negative values of o_t. We show below that it is important to decompose the oil price change Δo_t into more and less persistent components, and that their impacts on the macroeconomy are different.

The literature on the oil price-real output relationship is too broad to be fully reviewed here, so we focus in this section on briefly describing studies which are closely related to this paper. (See Hamilton (2008) for a recent survey.) Hamilton (1988) provides a theoretical framework for why the relationship between oil price and output is asymmetric: When the growth rate of oil price goes up, durable consumption growth drops, as consumers choose to postpone their purchases. But when the growth rate of the price of oil goes down, durable consumption growth does not necessarily rise. Mork (1989) provides evidence that allowing for this asymmetry – i.e. for different coefficients on price increases than on price decreases – is important. In particular, he finds that price decreases have little impact on US real output.

Hooker (1996), however, finds that lagged oil price changes do not explain current output growth after 1973, which suggests that either the relationship is unstable or, equivalently, that his model specification is problematic. Even allowing for asymmetry in the relationship, he still finds a poor fit using data subsequent to 1986. More recent studies focus on allowing for different forms of nonlinearity: Hamilton (1996) suggests that an oil price increase needs to exceed a threshold in order to have an impact; Ferderer (1996) and Jo (2012) argue that oil price volatility matters; Lee, Ni and Ratti (1995) suggest that an oil price increase needs to be unexpected; and Davis and Haltiwanger (2001) consider price changes sufficiently large as to make the price of oil surpass its previous five-year average. Hamilton (2003) uses a flexible functional form and confirms that both asymmetry and the "surprise" element are important. Finally, recent studies by Cunado and Perez de Gracia (2003), Jiménez-Rodríguez and Sánchez (2005) and Jiménez-Rodríguez (2009) also focus on the (asymmetric) nonlinearity in the relationship.

Kilian (2009) uses a structural VAR model to distinguish oil price movements that are induced by structural demand or supply shocks. These two shocks are shown to have different effects on income growth. In particular, a positive demand shock can drive up both income and oil price. The conclusion of their paper is that, during periods when both types of shocks are present, it is problematic to treat oil price changes as exogenous and only consider one-directional causality from oil price to income.[1]

Miller and Ni (2011) model the deviations from trend – deterministic and stochastic – of the oil price (o_t) as the sum of two change series: a component Δo_t^F ("updates to the long-term average price") obtained from an oil market model in levels, and a component Δo_t^U (essentially defined as the remainder of Δo_t) and interpreted as the "unanticipated" portion of Δo_t. In essence, one could say that they use an unobserved-components model – similar to a Beveridge-Nelson (1981) decomposition – to extract these two updates to a latent stochastic trend in o_t and find that these two components have different impacts on future real GDP growth.

The present paper statistically decomposes Δo_t in a new way, along a different dimension than the decomposition considered by Kilian (2009) and Miller and Ni (2011): here the impact

[1] Ordinary spectral regression yields inconsistent parameter estimates where feedback is present; the moving-window technique used here allows for possible feedback by decomposing oil price changes using an effectively one-sided filtering. This is described in the latter portion of Section 3.1 below.

on quarterly real growth is examined based on three components of Δo_t, each of which corresponds to fluctuations of a clear-cut level of persistence.[2] One component contains all sample variation in Δo_t identifiable as either a smooth nonlinear trend or as stochastic fluctuations with a persistence level corresponding to frequency components with a periodicity larger than four years. The second component comprises all sample variation in Δo_t with a persistence level corresponding to frequency components with a periodicity greater than one year and less than or equal to four years. And the third component comprises all sample variation in Δo_t with a persistence level of a year or less. These three components can be said to 'partition' the sample variation in Δo_t into these three persistence levels, as they are constructed so as to sum up to the original data on Δo_t. Hence, by replacing Δo_t in Equation (1) by a linear form in these three components, it is straightforward to decompose the real-growth impact of Δo_t by persistence level.

Section 2 describes the international data used here in estimating both Equation (1) and variations on it allowing for distinct (and asymmetric) responses to the components of Δo_t, partitioned into the three persistence levels discussed above. Section 3 describes and motivates the way in which this partitioning is done, based on frequency domain regression methods developed in Ashley and Verbrugge (2007) and Ashley, Tsang and Verbrugge (2012). Results on real GDP growth rates for the seven countries examined here are discussed in Section 4.

2. Data Description

Output growth (Δy_t) is the quarterly growth rate of real GDP for Australia, Canada, France, Japan, Norway, the United Kingdom and the United States; Norway is included so that the analysis also considers a net oil exporter.[3] This constitutes a representative sample of developed countries for which ample Δy_t data are available, omitting Germany due to its reunification

[2] See also Wei (2012), there the focus is solely on Japan.

[3] See the US Energy Information Administration website at http://www.eia.gov/countries/index.cfm?topL=exp for net oil export data on these countries. Canada and/or the UK are considered net oil exporters by some authors – e.g., Jiménez-Rodríguez and Sánchez (2005) – and this is, to one degree or another, the case for portions of the sample period considered here. In our empirical results Canada and the UK appears to 'behave' in the same way as do the countries which are clearly net oil importers, whereas the results for Norway are significantly distinct. Therefore, the grouping made at this point is used below, without substantial further comment.

during the period considered here.[4] All growth rates series are obtained from the St. Louis FRED database in annualized and seasonally adjusted form.

The oil price change series (Δo_t) used here is the annualized quarterly growth rate (in current US dollars) of the average of the of UK Brent Light, Dubai Medium and Alaska NS Heavy spot prices, all extracted from the International Monetary Fund's International Financial Statistics database.[5] The availability of each series and the sample period used for each in the empirical analysis are reported in Table 1a.

A time plot of Δo_t using data from 1952Q2 to 2011Q2 is given in Figure 1a. The impact of a number of political and economic events during this period is evident in the graph: the Arab oil embargo in 1973, the Iran-Iraq War in 1980, and the Persian Gulf War in 1990 all coincide with large spikes in the oil price growth rate series. Also evident is a large oil Δo_t drop during the collapse of OPEC around 1986 and an even larger drop in 2008, the latter of which clearly corresponds to the global recession of that year. Thus, as noted in Kilian (2009), substantial feedback (as distinct from unidirectional Granger causality) is likely present in the relationships between $\Delta o_t$and Δy_t considered here.[6]

A prominent feature of Figure 1a is the infrequency and small size of the fluctuations in Δo_t early in the available data set. Because of this, and because of the singular nature of the events in world oil markets in the early 1970's, the sample actually used here is truncated to begin in 1976Q1. Similarly, because the global macroeconomic fluctuation in 2008 so severely impacted both world oil markets and all seven national economies, the observations subsequent to 2007Q4 are also dropped from consideration, so as to prevent this event from having an inordinate impact on our results.[7] The data on Δo_t for the resulting sample period (1976Q1 to

[4] See Berument, Ceylan and Dogan (2010) for an analysis for the Middle Eastern and North Africa countries.
[5] Using oil price in constant dollars or using the West Texas Intermediate (WTI) spot price yield almost identical results. The oil price series are monthly, so we use the last month of each quarter to construct quarterly oil price series; using the average oil price over all three months also yields very similar results, which are available upon request. For simplicity (and because the analysis is in any case done entirely at a quarterly level), the windowed frequency decomposition described in Section 3 is applied to the quarterly oil price data.
[6] The possibility of feedback in a time-series relationship invalidates most frequency-domain-based regression methods. Notably, the approach used here is not vulnerable in this regard, because it is based entirely on one-sided filtering; this point is further discussed in Section 3 below.
[7] Analogous results additionally including the later sample data are collected in an appendix available from the authors, but are notably less interpretable: this single set of fluctuations in 2008 leads to a number of apparently-significant coefficient estimates with perverse signs. Ordinarily, one might consider allowing for the 2008

2007Q4) is plotted in Figure 1b and still contains ample sample variation. Finally, note (in Table 1a) that the data for France and Japan starts later on – in 1980Q1, when their real output growth series begin.

3. The Frequency-Dependence Approach

3.1 Description of the Method

The technique of modeling frequency dependence used here was originally developed by Tan and Ashley (1999a and 1999b) and further developed by Ashley and Verbrugge (2009).[8] The Ashley-Verbrugge approach is uniquely well-suited to the present application because – unlike other methods based on Fourier frequency decompositions – it is still valid in the presence of feedback in the relationship. It also does not require any *ad hoc* assumptions as to which frequencies correspond to a "business cycle" frequency band, etc.[9]

The remainder of this section briefly describes the procedure used here to decompose a time series (Δo_t, in the present instance) into frequency components. To simplify the notation, in this section the dependent variable is denoted as y_t and the vector of explanatory variables is denoted as X_t. We begin with the usual multiple regression model:

$$y = X\beta + u, \qquad u \sim N(0, \sigma^2 I) \tag{2}$$

where y is $T \times 1$, X is $T \times K$ and u is a $T \times 1$ vector of errors.[10] Now define a $T \times T$ matrix A, whose $(s,t)^{th}$ element $a_{s,t}$ is:

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turbulence using dummy variables. That option is essentially infeasible here because the decomposition of Δo_t into frequency components uses 16-quarter moving windows so as to yield a one-sided filtering – see Section 3 for details. Because these extraordinary fluctuations are so close to the end of the data set available, it seemed preferable to present results on a truncated sample rather than replace these data artificially with interpolated values.

[8] The idea of regression in the frequency domain can be traced back to Hannan (1963) and Engle (1974) – see Ashley and Verbrugge (2009) for details.

[9] The allowed frequencies are aggregated in the results presented here into three frequency bands primarily for expositional clarity; disaggregated results are available from the authors.

[10] Note that, while linear in β, Equation (2) is sufficiently general as to subsume a nonlinear (e.g. threshold autoregressive) or a cointegrated relationship.

$$a_{s,t} = \begin{cases} \left(\frac{1}{T}\right)^{\frac{1}{2}}, \quad for\ s = 1 \\ \left(\frac{2}{T}\right)^{\frac{1}{2}} \cos\left(\frac{\pi s(t-1)}{T}\right), \quad For\ s = 2,4,6,\dots,(T-2) or\ (T-1). \\ \left(\frac{2}{T}\right)^{\frac{1}{2}} \sin\left(\frac{\pi(s-1)(t-1)}{T}\right), \quad For\ s = 3,5,7,\dots,(T-1) or\ T. \\ \left(\frac{1}{T}\right)^{\frac{1}{2}} (-1)^{t+1}, \quad For\ s = T\ if\ T\ is\ even; otherwise\ this\ row\ is\ omitted. \end{cases} \tag{3}$$

This matrix embodies what is known as the "finite Fourier transform." It can be shown that A is orthonormal, i.e. $AA^t = I$.

Pre-multiplying the regression model in Equation (2) by A yields:

$$Ay = AX\beta + Au \rightarrow y^* = X^*\beta + u^*, \qquad u^* \sim N(0, \sigma^2 I) \tag{4}$$

While the dimensions of (y^*, X^*, and u^*) are the same as those (y, X, and u), the T components of y^* and u^* and the T rows of X^* now correspond to frequencies (denoted by index $s = 1 \dots T$) instead of to the time periods, $t = 1 \dots T$.[11]

The T frequency components corresponding to the j^{th} column of the X^* array are partitioned into $m = 1 \dots M$ frequency bands by means of M "dummy variables" $D^{*1,j}, \dots, D^{*M,j}$, each of which is a vector of dimension $T \times 1$. Each element of $D^{*m,j}$, the dummy vector for frequency band m, is defined to be equal to the corresponding element of vector X_j^* if that

[11] Reference to Equation (3), however, shows that s equal to two and three both refer to the same frequency – because there is both a "cosine" and a "sine" row in A. Similarly, s equal to four and five also both refer to the same frequency, and so forth. Thus, s runs from $1 \dots T$ and does index frequencies, but there are only $T/2$ distinct frequencies. The error terms u and u^* are different from each other but they are distributed identically because A is orthonormal.

element corresponds to a frequency in band *m*; otherwise $D^{*m,j}$ equals zero.[12] The regression model of Equation (4) can then be written as:

$$y^* = X^*_{!j}\beta_{!j} + \sum_{m=1}^{M} \beta_{j,m} D^{*m,j} + u^* \tag{5}$$

where $X^*_{!j}$ is the X^* matrix with the j^{th} column deleted and $\beta_{!j}$ is the β vector with the j^{th} component deleted. If the j^{th} component of β in the regression model is not frequency dependent, then $\beta_{j,1} = \cdots = \beta_{j,M}$.[13]

Because the dependent variable y^* in Equations (4) and (5) is in the frequency domain, and therefore more challenging to interpret, we pre-multiply both sides of Equation (5) by the inverse of A (which is simply its transpose), to obtain:

$$y = X_{!j}\beta_{!j} + \sum_{m=1}^{M} \beta_{j,m} D^{m,j} + u \tag{6}$$

The dependent variable, y, and the $K - 1$ columns of X remaining in $X_{!j}$ in Equation (6) are identical to those in the original model of Equation (2); the only difference is that the j^{th} variable in the original model is now replaced by M new explanatory variables $D^{1,j} \ldots D^{M,j}$, each with its own coefficient. Each of these M variables can be viewed as a bandpass-filtered version of the j^{th} column of the original X matrix.

One important property of this decomposition of the explanatory variable X_j into the M variables $D^{1,j} \ldots D^{M,j}$ is that these filtered components add up precisely to the original variable. That is: $X_j = D^{1,j} + \cdots + D^{M,j}$. Thus, to test if there is frequency dependence in the effect of the j^{th} regressor on y, one need only test the null hypothesis that $\beta_{j,1} = \cdots = \beta_{j,M}$.

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[12] The asterisk is included in the superscript of $D^{*m,j}$ to emphasize that elements of this vector refer to frequency rather than to time periods; the "*j*" is included so as to specify that components of this dummy variable are equal to either zero or to elements of the vector X^*_j.

[13] If, as is not the case here, one wants to allow for the possibility that β_l also varies across the *M* frequency bands for some $l \neq j$, then dummy variables $D^{*1,l} \ldots D^{*M,l}$ could be defined in an analogous manner.

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As with other Fourier transformation based band-pass filters, however, this decomposition of X_j based on the A transformation mixes up past and future values. Consequently – as is shown in Ashley and Verbrugge (2007) – the M frequency component vectors $D^{1,j}$ ... $D^{M,j}$ are correlated with the model error term u if there is feedback between y and X_j, leading to inconsistent OLS estimators for $\beta_{j,1}$ ... $\beta_{j,M}$ in Equation (6). This is a particularly compelling issue in the present context in view of the Kilian (2009) result – alluded to in Section 1 – that feedback is likely present between Δy and Δo. Ashley and Verbrugge (2007, 2009) eliminate this problem by modifying the procedure so as to yield a solely one-sided transformation of the data. In particular, they apply the decomposition procedure described above to just the data on the j^{th} explanatory variable which lies inside a small moving window, keeping only the most recent values of the M frequency components $D^{1,j}$ ... $D^{M,j}$ calculated for this particular window. This modification implies that the $D^{1,j}$ ... $D^{M,j}$ series actually used in estimating Equation (6) are the result of a one-sided rather than a two-sided band-pass filtering, eliminating this possible feedback-induced inconsistency in the OLS estimators of $\beta_{j,1}$ ... $\beta_{j,M}$. Note that each window yields one set of observations on $D^{1,j}$ ... $D^{M,j}$; thus, there are $T - M$ observations available for use in estimating Equation (6).

A good feature to this windowing is that the result of the first row of this, now much smaller, A matrix operating on a subset of X_j produces a "zero frequency" first component for X_j^* which is the sample mean of X_j using only the data from this window. Thus, the first component of $D^{1,j}$ is simply a moving average of the data on X_j as the window passes through the sample data, modeling in this way any (possibly nonlinear) smooth trend in the data, corresponding to a time-evolving estimate of the relevant agents' perceptions as to the most-persistent ("permanent") component of X_j.

A second feature of this windowing is that the lowest non-zero frequency which can be resolved (corresponding to rows two and three of the A matrix) corresponds to fluctuations in X_j with a reversal-period equal to the window length. This implies that any frequency components in X_j with a period larger than this window length are going to be indistinguishable from the deterministic moving-average trend corresponding to the "zero frequency" first component. On the other hand, a window of length, say 16 quarters, consumes 15 observations at the beginning of the sample, which would otherwise be available for parameter estimation.

In fact, the window size is chosen to be sixteen quarters in length for the present application; this turns out to be sufficiently long that the results are not sensitive to this choice, but one must bear this indistinguishability in mind when interpreting the results. In particular, with the 16-quarter window used here, fluctuations with a "reversal period" in excess of four years long are not distinguishable from the moving average trend component: both are included in the decomposition of X_j as part of its "zero-frequency" component. Section 3.2 below provides more detail (and intuition) on the meaning of the frequency components in the context of a simple example with a ten-period window.

In addition, when decomposing X_j using fairly short windows, one must deal with the standard problem of "edge effects" near the window endpoints. Following Dagum (1978) and Stock and Watson (1999), this problem is resolved by augmenting the data for each window with projections for one or two time periods. In the results quoted below, each sixteen-quarter window uses fifteen quarters of sample data and one projected value (for the sixteenth quarter) to produce the frequency components for the current period, i.e. for the fifteenth quarter of the window. The projection model used here for forecasting this last (projected) value is an AR(1) model estimated over the first fifteen sample observations in the window. (Using projections for two quarters at the end of each window yielded essentially identical results: these were obtained using fourteen actual sample values in each window and using an AR(1) model estimated over these fourteen observations to project the last two values. In that case the resulting frequency components for X_j correspond to the fourteenth window time period.) The $\beta_{j,1} \ldots \beta_{j,M}$ estimates and inference results are generally not sensitive to either the choice of the projection length (so long as it is small, but at least equal to one quarter), nor to the order of the projection model used.[14]

[14] The partitioning of X_j into frequency components as described above might appear to be a bit complicated, but it is very easy in practice, as readily-usable Windows-based software is available from the authors which inputs the sample data on X_j, the window length, the number of projections used in the window, and the order of the autoregressive model used in making the projections; it outputs the M frequency components – $D^{1,j} \ldots D^{M,j}$ – as a comma delimited spreadsheet file. An ideal projection model would utilize all information available at time t, but the AR(p) projection model specification makes the implementing software simpler and easier to use. Fortunately, the estimates of $\beta_{j,1} \ldots \beta_{j,M}$ are not very sensitive to exactly how the projections are done.

With a window size of sixteen quarters, one can decompose the oil price growth rate (Δo_t) into at most nine components.[15] It is possible to use all nine components in the regression. But in order to make the results easier to interpret (as well as to economize on the degrees of freedom), we regroup the nine components into three: a *low-frequency* component Δo_t^L, which contains the components of Δo_t corresponding to fluctuations with periods of more than sixteen quarters (i.e. the zero-frequency or "trend" component); a *medium-frequency* component Δo_t^M, which contains the components of Δo_t corresponding to fluctuations with periods larger than four quarters but less than sixteen quarters (i.e. the second, third and fourth possible frequencies); and a *high-frequency* component Δo_t^H, which contains the components of Δo_t corresponding to fluctuations with periods of four quarters or less (i.e. the fifth through ninth possible frequencies). This grouping of frequency components is summarized as follows:

Row of *A*	1	2, 3	4, 5	6, 7	8, 9	10, 11	12, 13	14, 15	16
Period of Fluctuation	> 16 quarters	16 quarters	8 quarters	5.33 quarters	4 quarters	3.2 quarters	2.67 quarters	2.29 quarters	2 quarters
Group	Low-freq. component Δo_t^L	Medium-freq. component Δo_t^M			High-freq. component Δo_t^H				

These three grouped components still add up to the original series, Δo_t. The low-frequency component contains the fluctuations in the oil price growth rate which appear to last more than four years, the high-frequency component contains fluctuations that appear to last one year or less; and the medium-frequency component contains everything in between.[16]

These three components are plotted in Figure 2, for a partitioning using one projection quarter in each window. (Instead using two projection quarters in each window yields virtually

[15] Reference to the definition of the *A* matrix in Equation (3) with *T* set to sixteen clarifies this statement. The first frequency component (from the first row of *A*, with *s* equal to one) is just the sample mean for this window; it corresponds to a frequency of zero. The second and third components both correspond (via the second row in *A* involving cosines and the third row in *A* involving sines) to the first non-zero frequency, the fourth and fifth components both correspond to the second non-zero frequency. And so forth, leading up to the fourteenth and fifteenth components – from the fourteenth and fifteenth rows of *A*, respectively – which both correspond to the seventh non-zero frequency. Finally (since sixteen is even) there is only one row of the A matrix for the largest (*T*/2 or eighth) non-zero frequency component – leading to nine possible frequencies in all.

[16] The explanatory variable Δo_t is not highly persistent. If it had been, then any trend-like component within the window would have substantial effects on the "medium frequency component". In such cases one might want to include some of the smaller non-zero frequencies in the "low frequency component."

identical plots and very similar regression results.[17]) These components would be precisely orthogonal to one another except for the use of a moving window. As explained above, this windowing ensures that the results are not adversely impacted by any feedback which might be present in the output versus oil-price relationship; it also allows for graceful nonlinear trend removal (in the zero-frequency component of Δo_t) via the windowed moving average. The sample correlations between these three components of Δo_t are still quite small, however: $\widehat{corr}(\Delta o_t^L, \Delta o_t^M)$= 0.037, $\widehat{corr}(\Delta o_t^L, \Delta o_t^H)$= -0.038, and $\widehat{corr}(\Delta o_t^M, \Delta o_t^H)$= 0.112. Note that all three components (as one would expect) display significant sample variation, but that the time variation in the low-frequency component is smoother than that of the medium-frequency component, which in turn is smoother than that of the high-frequency component.

3.2 The Appeal of this Frequency-based Approach to Disaggregation by Persistence Level

The objective of the partitioning of an explanatory variable time series – whether it is called X_j to be more parallel to other possible regressors or called Δo_t so as to make it more specific to the oil price model of Equation (1) – is not the band-pass filtering *per se*. Rather, the point of decomposing Δo_t into the components Δo_t^L, Δo_t^M, and Δo_t^H is entirely to make it possible to separately estimate the impact of fluctuations in Δo_t of distinctly different persistence levels on the growth rate of real output (y_t) and to thereby allow inferences to be made concerning these differential impacts.

No representation is made here that the band-pass filtering proposed above is in some sense "optimal" – e.g., as in Koopmans (1974) or Christiano and Fitzgerald (2003). Nevertheless, our method of decomposing a time series X_j into M frequency components has several very nice properties, which make it overwhelmingly well-suited to the present application:

1) The M frequency components which are generated from X_j by construction *partition* it: that is, these M components add up precisely to the original observed data on X_j. This makes estimation

[17] The sample correlations between the components obtained using two instead of one projection quarter are 0.936, 0.999, and 0.989 for the low, medium, and high-frequency bands, respectively; this correlation (while still high) is a bit lower for the low-frequency band because the zero-frequency component is in that case a fourteen-quarter moving average instead of a fifteen-quarter moving average.

and inference with regard to frequency (or ‘persistence,’ its inverse) dependence in the coefficient β_j particularly straightforward.

2) Due to the moving windows used, this particular way of partitioning of X_j into these M frequency components is via a set of entirely *backward-looking* (i.e., “one-sided”) filters. This feature is essential to consistent OLS estimation of the coefficient β_j in the – here, quite likely – circumstance where there is bi-directional Granger-causality (feedback) between y and X_j.

3) And, finally, this partitioning of X_j into frequency components is not just mathematically valid and straightforward: it is also intuitively appealing.[18] The next section illustrates this with a simple example.

3.3 An Explicit Example with a Short Window

An example with a window ten periods in length illustrates the sense in which the frequency components define above are extracting components of X_j of differing levels of persistence.[19] Table 2 explicitly displays the multiplication of the matrix A – whose $(s,t)^{th}$ element is given in Equation (3) – by the ten-component sub-vector of X_j corresponding to a window beginning in period twenty one.

The first component of this matrix product corresponds to what one might call the “zero-frequency” component of this subsample of X_j. Note that the “Period” column in Table 2 is essentially just the reciprocal of the frequency corresponding to the sine or cosine used in the

[18] The Christiano-Fitzgerald (2003) bandpass filter could in principle be repeatedly applied to the X_j data in a given window; the filter could then be iteratively applied to the remainder from the previous repetition, only with a different lower bound for the frequency band at each iteration. This rather unwieldy procedure would yield a decomposition of the explanatory variable whose frequency components still add up to the original data on X_j. Note, however, that such a decomposition would be more complicated, less intuitively appealing, and in fact no more “optimal” than ours.

[19] A window ten periods in length is sufficiently large as to illustrate the point, while sufficiently small as to allow Table 2 to fit onto a single page; the actual implementation in Section 4 uses a window sixteen periods in length, so that the medium-frequency component can subsume fluctuations with a reversal period of up to four years.

corresponding row of the A matrix. The first entry in this column of Table 2, corresponding to a frequency of zero, is thus arbitrarily large.[20]

The first row of the A matrix is just a constant, so the operation of this row on the ten-vector sub-component of X_j is in essence just calculating its sample mean over these ten observations. Thus, the zero-frequency component of X_j is actually just a one-sided (or "real-time") moving-average nonlinear trend estimate. As noted earlier, this "zero-frequency" component is also subsuming any stochastic fluctuations in X_j at frequencies so low (periodicities or persistence so large) as to be invisible in a window which is only ten periods in length.

The next two rows of A (and hence the next two components of the matrix product) both correspond to a periodicity of ten quarters because in both cases the elements of the A row vary per a sine or cosine which completes one cycle ("period") in ten quarters. Thus, the second and third components of AX_j will both be small for any variation in X_j which basically reverses itself within a few quarters, whereas these two components will be large for any variation in X_j takes circa ten quarters to reverse itself – i.e., for variation in X_j which is "low-frequency."[21] In contrast, looking at the tenth row of the A matrix, it is evident that the inner product of this row with a slowly-varying X_j sub-vector will yield only a small value for the tenth component of AX_j, whereas an X_j sub-vector which corresponds to a high-frequency fluctuation – i.e., which reverses in just a quarter or two – will contribute significantly to the tenth component of AX_j.

Thus, the first rows of the A matrix are distinguishing and extracting what are sensibly the "low-frequency," or "large period," or "highly persistent" – or 'permanent' – components of this ten-quarter X_j sub-vector. And, concomitantly, the last rows of the A matrix are distinguishing and extracting what are sensibly the "high-frequency," or "small period," or "low persistence" – or 'temporary' – components of this X_j sub-vector.

[20] Technically, the frequency is 2π divided by the period of the corresponding sine or cosine, but that detail is not important here.
[21] The first few lowest frequency components model any trend-like behavior in the data within the window.

4. Results and Discussion

Re-specifying Equation (1) for each country so as to explicitly allow for asymmetry in the response to oil price growth rate fluctuations yields:

$$\Delta y_{t+1} = \alpha + \gamma_1 \Delta y_t + \gamma_2 \Delta y_{t-1} + \beta_+ \Delta o_t^+ + \beta_- \Delta o_t^- + u_t \tag{7}$$

This is essentially the standard specification in the literature, e.g., from Hamilton (1983) through Hamilton (2008).[22] Two lags of Δy_t are included so as to allow for serial correlation in the model errors in all seven countries; for several countries inclusion of just one lag yields model fitting errors with statistically significant serial correlation. Output growth in Equation (7) can respond to an increase in the growth rate of the oil price (Δo_t^+) differently than to a decrease (Δo_t^-), but this specification does not allow for differential responses to fluctuations in Δo_t with differing levels of persistence.[23]

Using the methodology described in Section 3 above, the Δo_t time series is here further decomposed by persistence level into low, medium, and high-frequency components: Δo_t^L, Δo_t^M, and Δo_t^H. This yields the model specification:

$$\begin{aligned}\Delta y_{t+1} &= \alpha + \gamma_1 \Delta y_t + \gamma_2 \Delta y_{t-1} + \beta_{+,L} \Delta o_t^{+,L} + \beta_{+,M} \Delta o_t^{+,M} + \beta_{+,H} \Delta o_t^{+,H} \\ &+ \beta_{-,L} \Delta o_t^{-,L} + \beta_{-,M} \Delta o_t^{-,M} + \beta_{-,H} \Delta o_t^{-,H} + u_t\end{aligned} \tag{8}$$

Each frequency component of Δo_t is itself separated into its positive and negative values in Equation (8). Thus, for example, $\Delta o_t^{+,L}$ equals Δo_t^L in each period for which Δo_t^L is non-negative and is zero otherwise, whereas $\Delta o_t^{-,L}$ equals Δo_t^L in each period for which Δo_t^L is negative.

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[22] It is particularly reasonable to model this relationship in terms of growth rates rather than levels, as it is not possible to reject the null hypothesis of unit root in either y_t or o_t on an augmented Dickey-Fuller (ADF) test for any of these countries. This standard specification in terms of growth rates is almost certainly preferable even if these ADF tests are incorrectly failing to reject, as both levels series are clearly quite persistent – see discussion in the Appendix, available from the authors. Also, the Johansen test provides no evidence for cointegration between $y_t$and o_t; for this reason (only) an error-correction term is not included in Equations (7) or (8); using a different test (and a different sample period), Ghosh, Varvares, and Morley (2009) finds cointegration in the 4-vector (y_t, o_t, hours, and productivity) for the US.

[23] The time series Δo_t^+ is defined to equal in Δo_t in every time period for which the Δo_t is greater than or equal to zero; the time series Δo_t^- is defined analogously. The value of Δo_t is zero in only six instances during the course of the samples on the seven countries.

Thus, this model specification allows for both asymmetry and for differential responses to oil price growth rate fluctuations with varying levels of persistence.[24]

These two model specifications are estimated via OLS using sample data from each of the seven countries – Australia, Canada, France, Japan, Norway, United Kingdom and the United States – using (as described in Section 2) the sample period 1976Q1 to 2007Q4 in each case, except starting in 1978Q1 for Norway and in 1980Q1 for Australia and France; this sample yields 120 observations for Norway, 112 observations for Australia and France and 128 observations for the rest of the countries. Heteroscedasticity and strong cross-country cross-correlations in the model errors are allowed for by treating the seven regression equations as a system of seemingly unrelated regressions (SUR) and thereby re-estimating the regression coefficient standard errors.[25] Serial correlation in the model errors (u_t) is eliminated by including a pair of lagged dependent variables in the model for each country.

This paper is actually all about the model specification of Equation (8), allowing for frequency (persistence) dependence as well as asymmetry in the coefficient on Δo_t. For comparison, however, we first present results on the model specification of Equation (7), which allows for asymmetry only; these results are reported in Table 3. For four of the countries (Australia, Canada, the UK, and the US) there is no real evidence for rejecting either H_o: $\beta_+ = 0$ or H_o: $\beta_- = 0$; that is, neither an increase in the oil price growth rate ($\Delta o_t > 0$) nor a decrease in the oil price growth rate ($\Delta o_t < 0$) appears to have any impact on the real output growth rate for these four countries in this model specification. The β_- estimate for France is statistically significant and has the (negative) sign expected for an oil-importing country – that is, a drop in

[24] Because of the lagged dependent variables in Equation (8) – included so as to ensure that the model error u_t is serially uncorrelated – the coefficients on the various components of Δo_t should be interpreted as 'impact multipliers' rather than as 'long-run multipliers.' There is no contradiction, however, in positing that the current impact of a low-frequency ('permanent') fluctuation in Δo_t differs from the current impact of a high-frequency ('temporary') fluctuation in Δo_t. Also, cointegration is not usually considered for specifications in this literature, but (as noted in Footnote #22 above) the methodology used here does not preclude inclusion of an error-correction term in the model specification; here no error-correction term was found to be necessary.

[25] This was accomplished using use the Stata *suest* post-estimation command. This technique exploits the cross-equation model error correlations, so as to obtain better estimators of the coefficient standard errors; it also makes it possible to test joint null hypotheses involving coefficients from more than one country. We are not, however, using Zellner's SURE estimator; Zellner's estimator can increase parameter estimation efficiency, but requires an assumption of homoscedastic model errors for each country. Here the *suest* routine corrects the standard error estimates with respect to within-country heteroscedasticity of any form.

the oil price growth rate enhances real growth. On the other hand, the β_+ estimate is not statistically significant and the null hypothesis that $\beta_+ = \beta_-$ cannot be rejected.

In contrast, for Japan and Norway, either β_+ or β_- is statistically significant, but has a positive sign. One would ordinarily expect that a change in the oil price growth rate would have the opposite impact on an oil-importing country's real output growth rate; thus, all of the coefficients on oil price growth rates are expected to be negative, at least for countries which are net importers of oil. Of course, Norway is actually a major net exporter of oil,[26] so this statistically significant and positive value for β_+ is not anomalous for Norway. But the statistically significant positive estimate of β_- for Japan is hard to rationalize. Below, however, we show that this anomaly disappears once one controls for differing persistence levels in the Δo_t^+ and Δo_t^- values for Japan.

The last three lines of Table 3 give the *p*-values at which – jointly over all seven countries – one can reject the null hypothesis of symmetry ($\beta_+ = \beta_-$), the null hypothesis that positive values of Δo_t have no impact ($\beta_+ = 0$), and the null hypothesis that negative values of Δo_t have no impact ($\beta_- = 0$). These joint hypothesis tests are making use of the fact that the seven country-specific regression models are being treated as a system of equations in estimating the coefficient standard errors.[27] These three entries in Table 3 also report *p*-values for the analogous joint hypothesis tests in which the portions of the joint null hypothesis relating to Japan and Norway (the two countries with individually significant coefficient estimates with 'perverse' sign) are omitted. The symmetry null hypothesis and the null hypothesis that positive values of Δo_t have no impact ($\beta_+ = 0$) are no longer rejected once Japan and Norway are omitted, indicating that the results on these two countries are driving those two results. The null hypothesis that negative values of Δo_t have no impact ($\beta_- = 0$) can still be rejected at the 1% level because of the strong result to this effect in the regression model for France.

We next turn to the results, which are the point of present work, based on the specification given in Equation (8). These results are summarized in Table 4, and disaggregate

[26] Norway imports little oil compared to the amount it exports. Canada also is a net exporter of oil, but it imports almost as much as it exports; for simplicity of exposition, Canada will be grouped with the oil-importing countries in the exposition below.

[27] This technique also exploits the positive cross-equation correlations in the model errors to yield notably smaller estimated coefficient standard error estimates; see Footnote #25 above.

the coefficients on Δo_t^+ and Δo_t^- into the three persistence levels defined in Section 3: a superscript of "*L*" indicates fluctuations with a period in excess of four years, or "low-frequency" fluctuations; a superscript of "*M*" indicates fluctuations with a period in excess of one year but less than or equal to four years, or "medium-frequency fluctuations"; and a superscript of "*H*" indicates fluctuations with a period of less than or equal to one year, or "high-frequency" fluctuations

The first thing to notice in the Table 4 results, now allowing β_+ and β_- to vary with the persistence level of the corresponding Δo_t^+ and Δo_t^- variables, is that the 'perverse' coefficient on Δo_t^- for Japan is no longer present. In fact, except for Norway, none of the coefficients on $\Delta o_t^{-,L},\ o_t^{-,M},\ \Delta o_t^{-,H},\ \Delta o_t^{+,L},\ o_t^{+,M},$ or $\Delta o_t^{+,H}$ is both statistically significant and positive. In this sense the richer model specification of Equation (8), disaggregating Δo_t^+ and Δo_t^- by persistence level, is already an improvement on the asymmetry-only model of Equation (7).

Because it is a major net exporter of oil, for Norway a statistically significant positive coefficient is not surprising. It is interesting, however, that this is the case only for $\beta_{+,M}$: the other coefficient estimates for Norway are either statistically insignificant or (for $\beta_{-,H}$) statistically significant and negative. These results suggest that Norway responds as an oil exporter would to a moderately-persistent increase in Δo_t, but that it responds as oil-importing country might to a drop in Δo_t which is seen as 'temporary' – i.e., which is expected to persist only a year or less. But Norway responds in this fashion only to oil price growth rate *drops*, whereas – as will be noted below – the oil importing countries respond in this way to low-persistence oil price growth rate *increases*.

Turning now to the Table 4 results on the other six (oil-importing) countries – i.e., omitting consideration of Norway – there are two ways to organize a summary of these results: by the sign of Δo_t and by the persistence level (*L*, *M*, or *H*) in Δo_t.

First, considering how the results vary with persistence level, note that there is no evidence for any impact of either $\Delta o_t^{+,M}$ or $\Delta o_t^{-,M}$ on Δy_t: evidently the growth rate in oil prices affects a country's real output growth rate only for either high-frequency ('temporary') fluctuations (of a year or less) or for low-frequency ('permanent') fluctuations more than four years in length.

Thus, frequency dependence is in fact a prominent feature in this relationship; this conclusion is amply borne out by the formal hypothesis test results quoted in Table 5. In particular, not that – while the "no-frequency-dependence" null hypothesis is not rejected for the six net-oil-importing countries individually – the evidence against this null hypothesis is compelling when the regression results for these six countries are appropriately combined to test the joint hypothesis for all six countries simultaneously.

Next, we focus on just the high-frequency oil price growth fluctuations (i.e., on the impact of either $\Delta o_t^{+,H}$ and $\Delta o_t^{-,H}$) in the six net oil-importing countries. Here there is strong evidence of asymmetry in the relationship: a high-frequency increase in the growth rate of oil prices has a statistically significant negative impact on real output growth in Canada, France, the UK, and the US – with weaker impacts of the same sign in Australia and Japan. In contrast, there is no evidence that a high-frequency decrease in the growth rate of oil prices has any impact on the real output growth rate in any of these six countries at even the 5% level.

There is also strong evidence for asymmetry in the low-frequency oil price growth fluctuations (i.e., on the impact of either $\Delta o_t^{+,L}$ and $\Delta o_t^{-,L}$) in the six net oil-importing countries, but of a more complicated form. In particular, for highly-persistent Δo_t fluctuations, there is a mixture of statistically significant results. Oil price growth rate *increases* ($\Delta o_t^{+,L}$) in Australia and the US have a negative impact on real output growth which is both statistically and economically strong, but the impact of oil price growth rate decreases ($\Delta o_t^{-,L}$) is not statistically different from zero in these two countries. Whereas – in France and the UK – it is low-frequency oil price *decreases* which have a statistically and economically significant impact (in the opposite direction) on real output growth.[28]

Again, the formal hypothesis testing results given in Table 5 confirm these asymmetry conclusions, in a statistically compelling way for the hypothesis tests which are formulated jointly over all six net-oil-importing countries.

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[28] We prefer to err on the side of under-interpreting the significance of the $\beta_{-,L}$ estimate for Japan because this one coefficient estimate was no longer statistically significant when two projection quarters were used (instead of one) in the moving window utilized for partitioning Δo_t into persistence components; our other results are qualitatively independent of such modeling choices.

5. Concluding Remarks

We find that a model specification for the growth rate in real output allowing only for asymmetry in the coefficient on the oil price growth rate (Δo_t) – i.e., Equation (7) – can mislead one into thinking that changes in Δo_t have either no statistically significant impact or a perverse effect on real output growth. For example, someone considering only the results in Table 3 might well conclude that positive values of Δo_t have no impact on real output growth in Australia, Canada, France, the UK and the US and that a negative value for Δo_t has a statistically significant impact with a perverse sign in Japan.

In contrast, our results in Table 4 show that – allowing for frequency dependence in the real output growth rate model, i.e., for varying responses to different levels of persistence in sample fluctuations in Δo_t^+ or Δo_t^- – yields results which are both statistically significant and economically explicable. Broadly, for the six countries (Australia, Canada, France, Japan, the UK and the US) which are not major oil exporters :

a) High-frequency ('temporary') Δo_t increases, which typically reverse within one to four quarters, depress real output growth rates, whereas high-frequency Δo_t decreases appear to have little impact on real output growth rates.

b) Mid-frequency Δo_t changes (in either direction), which typically reverse within one to four years, appear to have little impact on real output growth rates.

c) And low-frequency ('permanent') Δo_t changes have statistically and economically significant impacts – for positive Δo_t changes in Australia and the US, and for negative Δo_t changes in France, Japan, and the UK.

For the major net oil-exporting country in our sample (Norway), we see a different result: a mid-frequency increase in Δo_t actually increases the real output growth rate. Note that this result is consistent with the finding in Jiménez-Rodríguez and Sánchez (2005) that Δo_t^+ enters a model specification like Equation (7) with a significant positive coefficient. But our result is richer (and more nuanced) in that we find that this effect of an increase in Δo_t only pertains to oil price growth rate increases with a persistence in the range of more than one year but less than or equal to four years. Additionally, we find that – unlike the net oil importing countries – Norway responds to low-persistence decreases, rather than increases, in the oil price growth rate with a change in the real output growth rate of the opposite sign.

Overall, then, our results demonstrate the existence of a strong (and asymmetric) impact of oil prices on real output, once one appropriately allows for the differential impact of fluctuations in the oil price growth rate with differing levels of persistence. Controlling for the persistence level of oil price fluctuations not only leads to a more statistically adequate econometric formulation of the real output versus oil price relationship, but also yields interestingly interpretable economic results.

What are the mechanisms behind our reduced-form results? Do households and firms respond differently to oil price changes of different levels persistence? And if so, why? Does the central bank react differently as well? Our results point clearly to a need for a more structural theory, leading to a model capable of addressing these new questions.

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Table 1a: Data Availability

	Real GDP Growth Δy_t	Source	Number of Observations
Australia	1976Q1 – 2007Q4	FRED	126
Canada	1976Q1 – 2007Q4	FRED	126
France	1980Q1 – 2007Q4	FRED	109
Japan	1980Q1 – 2007Q4	FRED	109
Norway	1978Q1 – 2007Q4	FRED	117
United Kingdom	1976Q1 – 2007Q4	FRED	126
United States	1976Q1 – 2007Q4	FRED	126
	Oil Price Growth Δo_t	**Source**	
IMF Spot Oil Price	1976Q1 – 2007Q4	IFS	

Table 1b: Summary Statistics on the Oil Price Growth Rate and Its Components

	Δo_t	Δo_t^L	Δo_t^M	Δo_t^H
Mean	6.200	7.053	-0.194	-0.659
Median	0.962	5.493	-0.736	-0.073
Maximum	324.699	46.331	192.916	258.930
Minimum	-295.774	-26.990	-105.768	-168.081
SD	65.494	15.678	38.265	46.800
Skewness	0.304	0.257	0.979	0.598
Kurtosis	10.181	2.369	7.611	11.108

Table 2: A Ten-Period Window Example

Period	Matrix A										Data
∞	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	$X_j(21)$
10	0.45	0.36	0.14	−0.14	−0.36	−0.45	−0.36	−0.14	0.14	0.36	$X_j(22)$
10	0.00	0.26	0.43	0.43	0.26	0.00	−0.26	−0.43	−0.43	−0.26	$X_j(23)$
5	0.45	0.14	−0.36	−0.36	0.14	0.45	0.14	−0.36	−0.36	0.14	$X_j(24)$
5	0.00	0.43	0.26	−0.26	−0.43	0.00	0.43	0.26	−0.26	−0.43	$X_j(25)$
3.3	0.45	−0.14	−0.36	0.36	0.14	−0.45	0.14	0.36	−0.36	−0.14	$X_j(26)$
3.3	0.00	0.43	−0.26	−0.26	0.43	0.00	−0.43	0.26	0.26	−0.43	$X_j(27)$
2.5	0.45	−0.36	0.14	0.14	−0.36	0.45	−0.36	0.14	0.14	−0.36	$X_j(28)$
2.5	0.00	0.26	−0.43	0.43	−0.26	0.00	0.26	−0.43	0.43	−0.26	$X_j(29)$
2	0.32	−0.32	0.32	−0.32	0.32	−0.32	0.32	−0.32	0.32	−0.32	$X_j(30)$

Note: The first row of A time the data vector simply yields $1/\sqrt{T}$ times the sample mean of the data in this ten-period window. As the window moves through the data set, this operation extracts any, possibly nonlinear, trend as a moving average. Rows two and three take a weighted average of the window data, using smoothly-varying weights which take a full ten periods to reverse, so any fluctuation in window data that reverses in a couple of periods yields a small value. The product of row ten and the window data is essentially calculating five changes in the data which occur during the window period; a long, smooth variation in the window data yields a small value for this frequency component.

Table 3: Oil Price-Output Regression Allowing Only for Asymmetry

	Australia	Canada	France	Japan	UK	US	Norway
Increase in oil price growth rate: Δo_t^+	-0.005 (0.083)	-0.002 (0.005)	-0.002 (0.005)	-0.014** (0.006)	-0.008 (0.012)	-0.009 (0.006)	0.018** (0.008)
Decrease in oil price growth rate: Δo_t^-	0.008 (0.005)	-0.004 (0.016)	-0.008*** (0.002)	0.016*** (0.006)	0.001 (0.006)	0.002 (0.005)	-0.004 (0.008)
Adjusted R^2	0.000	0.219	0.223	0.037	0.054	0.066	0.079
H_o: $\beta_+ = \beta_-$ (Symmetry) [p-value]	0.293	0.737	0.305	0.001	0.514	0.241	0.091
H_o: $\beta_+ = \beta_-$ (Symmetry – Joint Test) [p-value]	0.000 (0.128, omitting Japan and Norway)						
H_o: $\beta_+ = 0$ (No impact from Δo_t^+) [p-value]	0.001 (0.157, omitting Japan and Norway)						
H_o: $\beta_- = 0$ (No impact from Δo_t^-) [p-value]	0.000 (0.006 , omitting Japan and Norway)						

Note: The regression model corresponds to Equation (7): $\Delta y_{t+1} = \alpha + \gamma_1 \Delta y_t + \gamma_2 \Delta y_{t-1} + \beta_+ \Delta o_t^+ + \beta_- \Delta o_t^- + u_t$. All variables are in annualized percentage changes. Figures in parentheses are estimated standard errors, using the Stata *suest* postestimation routine to account for both heteroskedasticity and contemporaneous cross-country correlations in the model errors; all hypothesis tests use these standard errors – see Footnote #25. P-values reported here are for two-tailed tests throughout. For brevity, the constant term and the estimated coefficients on the two lags of output growth are not reported.

Table 4: Oil Price-Output Regression Allowing for Both Asymmetry and Frequency Dependence

	Australia	**Canada**	**France**	**Japan**	**UK**	**US**	**Norway**
Increase in oil price growth rate, high-frequency: $\Delta o_t^{+,H}$	-0.010 (0.007)	-0.019*** (0.006)	-0.009** (0.004)	-0.013* (0.008)	-0.016** (0.008)	-0.019*** (0.007)	0.002 (0.009)
Decrease in oil price growth rate, high-frequency: $\Delta o_t^{-,H}$	0.018 (0.014)	0.007 (0.012)	-0.003 (0.004)	0.020 (0.014)	0.003 (0.010)	0.012 (0.009)	-0.024** (0.011)
Increase in oil price growth rate, medium-frequency: $\Delta o_t^{+,M}$	0.004 (0.015)	0.006 (0.009)	0.012 (0.009)	-0.011 (0.025)	-0.007 (0.024)	0.003 (0.008)	0.041** (0.018)
Decrease in oil price growth rate, medium-frequency: $\Delta o_t^{-,M}$	0.001 (0.016)	0.007 (0.012)	-0.007 (0.007)	0.014 (0.019)	0.023 (0.016)	-0.002 (0.015)	0.034 (0.023)
Increase in oil price growth rate, low-frequency: $\Delta o_t^{+,L}$	-0.067** (0.028)	-0.025 (0.024)	0.008 (0.015)	0.010 (0.033)	-0.031 (0.028)	-0.059** (0.030)	-0.070 (0.054)
Decrease in oil price growth rate, low-frequency: $\Delta o_t^{-,L}$	-0.004 (0.046)	-0.071 (0.046)	-0.050** (0.029)	-0.145** (0.070)	-0.096** (0.048)	-0.010 (0.035)	0.134 (0.091)
Number of observations	126	126	109	109	126	126	117

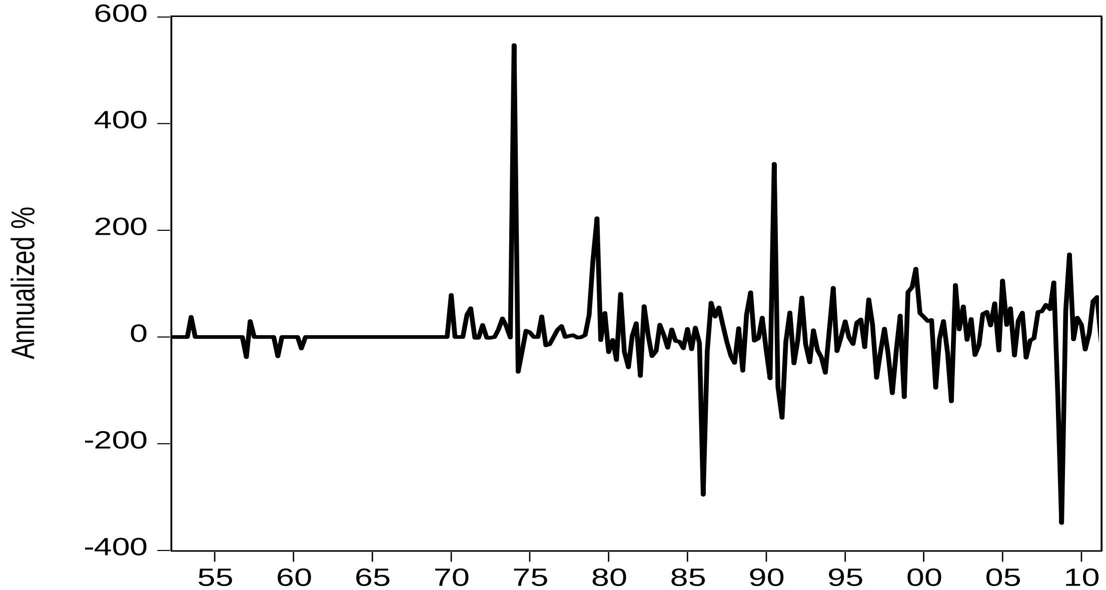
Note: The regression model corresponds to Equation (8): $\Delta y_{t+1} = \alpha + \gamma_1 \Delta y_t + \gamma_2 \Delta y_{t-1} + \beta_{+,L} \Delta o_t^{+,L} + \beta_{+,M} \Delta o_t^{+,M} + \beta_{+,H} \Delta o_t^{+,H} + \beta_{-,L} \Delta o_t^{-,L} + \beta_{-,M} \Delta o_t^{-,M} + \beta_{-,H} \Delta o_t^{-,H} + u_t$. All variables are in annualized percentage changes. Figures in parentheses are estimated standard errors, using the Stata *suest* postestimation routine to account for both heteroskedasticity and contemporaneous cross-country correlations in the model errors; all hypothesis tests use these standard errors – see Footnote #25. *P*-values reported here are for two-tailed tests throughout. For brevity, the constant term and the estimated coefficients on the two lags of output growth are not reported.

Table 5: Additional Hypothesis Tests for Oil Price-Output Regression Allowing for Both Asymmetry and Frequency Dependence (p-values)

<table>
<tr><th></th><th>Australia</th><th>Canada</th><th>France</th><th>Japan</th><th>UK</th><th>US</th><th>Norway</th></tr>
<tr><td>H_o: $\beta_{+,L} = \beta_{+,M} = \beta_{+,H}, \beta_{-,L} = \beta_{-,M} = \beta_{-,H}$ (Frequency Independence)</td><td>0.066</td><td>0.171</td><td>0.215</td><td>0.200</td><td>0.078</td><td>0.077</td><td>0.009</td></tr>
<tr><td>H_o: $\beta_{+,L} = \beta_{+,M} = \beta_{+,H}, \beta_{-,L} = \beta_{-,M} = \beta_{-,H}$ (Frequency Independence – Joint Test)</td><td colspan="7">0.000</td></tr>
<tr><td>H_o: $\beta_{+,L} = \beta_{+,M} = \beta_{+,H}$ (Frequency Independence, increases only)</td><td>0.058</td><td>0.144</td><td>0.199</td><td>0.772</td><td>0.765</td><td>0.061</td><td>0.132</td></tr>
<tr><td>H_o: $\beta_{+,L} = \beta_{+,M} = \beta_{+,H}$ (Frequency Independence – Joint Test)</td><td colspan="7">0.028</td></tr>
<tr><td>H_o: $\beta_{-,L} = \beta_{-,M} = \beta_{-,H}$ (Frequency Independence, decreases only)</td><td>0.747</td><td>0.362</td><td>0.278</td><td>0.056</td><td>0.108</td><td>0.644</td><td>0.039</td></tr>
<tr><td>H_o: $\beta_{-,L} = \beta_{-,M} = \beta_{-,H}$ (Frequency Independence – Joint Test)</td><td colspan="7">0.015</td></tr>
<tr><td>H_o: $\beta_{+,L} = \beta_{-,L}, \beta_{+,M} = \beta_{-,M}, \beta_{+,H} = \beta_{-,H}$ (Symmetry)</td><td>0.176</td><td>0.334</td><td>0.244</td><td>0.039</td><td>0.128</td><td>0.076</td><td>0.147</td></tr>
<tr><td>H_o: $\beta_{+,L} = \beta_{-,L}, \beta_{+,M} = \beta_{-,M}, \beta_{+,H} = \beta_{-,H}$ (Symmetry – Joint Test)</td><td colspan="7">0.000</td></tr>
<tr><td>H_o: $\beta_{+,L} = \beta_{+,M} = \beta_{+,H} = 0$ (No impact from $\Delta o_t^{+,L}, \Delta o_t^{+,M}, \Delta o_t^{+,H}$)</td><td>0.040</td><td>0.008</td><td>0.166</td><td>0.121</td><td>0.045</td><td>0.010</td><td>0.055</td></tr>
<tr><td>H_o: $\beta_{+,L} = \beta_{+,M} = \beta_{+,H} = 0$ (No impact from $\Delta o_t^{+,L}, \Delta o_t^{+,M}, \Delta o_t^{+,H}$ – Joint Test)</td><td colspan="7">0.000</td></tr>
<tr><td>H_o: $\beta_{-,L} = \beta_{-,M} = \beta_{-,H} = 0$ (No impact from $\Delta o_t^{-,L}, \Delta o_t^{-,M}, \Delta o_t^{-,H}$)</td><td>0.467</td><td>0.536</td><td>0.001</td><td>0.077</td><td>0.196</td><td>0.534</td><td>0.085</td></tr>
<tr><td>H_o: $\beta_{-,L} = \beta_{-,M} = \beta_{-,H} = 0$ (No impact from $\Delta o_t^{-,L}, \Delta o_t^{-,M}, \Delta o_t^{-,H}$ – Joint Test)</td><td colspan="7">0.000</td></tr>
</table>

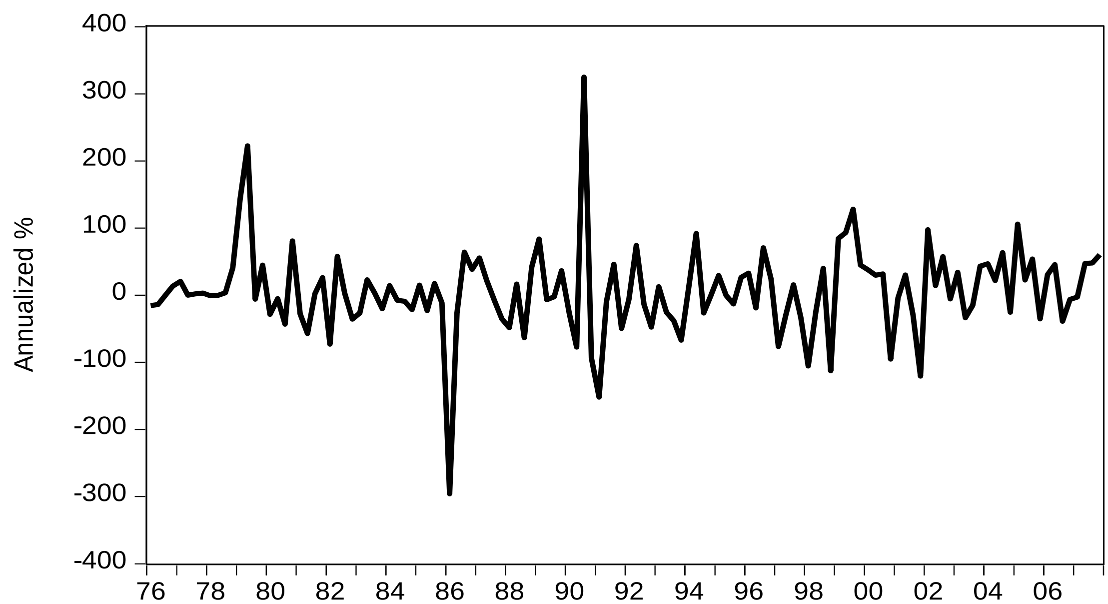
Note: See the note for Table 4.

Figure 1a: Annualized quarterly nominal oil price growth rate Δo_t (1952Q2 – 2011Q2)



Source: IMF International Financial Statistics database; see Section 2 for details.

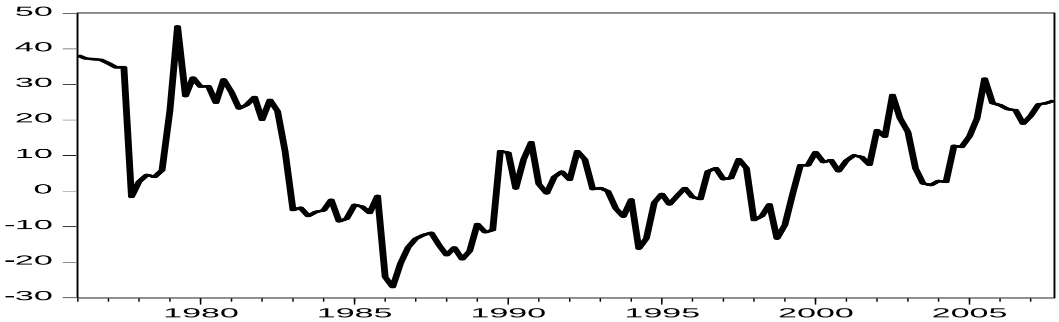
Figure 1b: Annualized quarterly nominal oil price growth rate Δo_t (1976Q1 – 2007Q4)



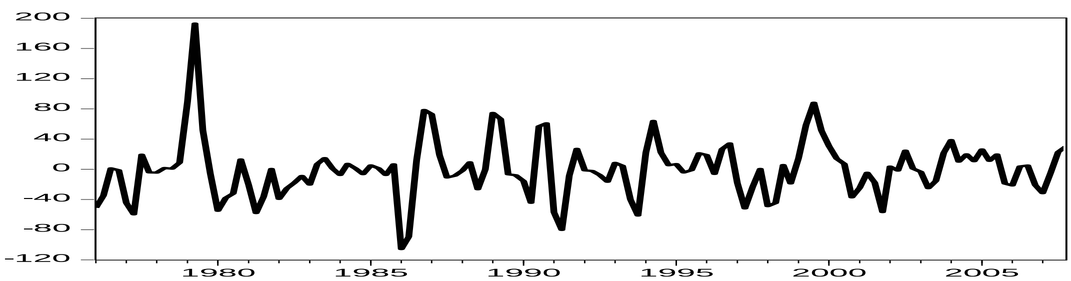
Source: IMF International Financial Statistics database; see Section 2 for details.

Figure 2: Frequency (Persistence) Components of Oil Price Growth Rate (Δo_t)

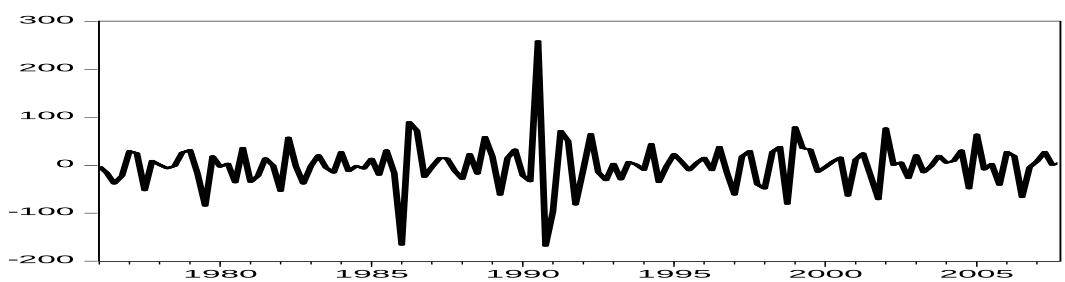
Low-frequency Component (> 4 Years)



Medium-Frequency Component (≤ 4 Years and > 1 Year)



High-frequency Component (≤ 1 Year)



Note: Δo_t is decomposed into three components. The low-frequency component has a period of fluctuations of more than 4 years, the medium-frequency component is equal or less than to four years and more than one year, and the high-frequency component is one year or less. The sum of the three components is exactly equal to the original oil price growth rate series plotted in Figure 1b.

Appendix: Modeling in Levels

Equation (7), as is fairly standard in this literature, specifies the relationship between real output and oil prices in terms of their growth rates. One could instead model the relationship between the levels (log-levels, presumably) of these time series. However, in view of the fact that both of these time series are either integrated – i.e., I(1) – or nearly so, we view modeling this relationship in terms of levels unwise.

The main point of this paper is that the strength (and even the sign) of the relationship between oil prices and real output can (and does) depend on the persistence of the fluctuations in oil prices. And we do in this paper explicitly examine the relationship between the growth rates of real output and the oil price at various frequency components, including a component at a frequency of zero. This zero-frequency component amounts to a moving average of recent values of the growth rates in the oil price; this component captures any slow, smooth changes in the mean growth rate of oil prices. In addition, we explicitly examine whether there is cointegration between the log-level series, and find that there is not. Had evidence for such cointegration materialized, however, the most appropriate response would have been to have included an error-correction term in the growth rate equation, consisting of the fitting errors from a levels model between these two series. The coefficient on that error-correction term – had it existed – would then have quantified something different from the zero-frequency coefficient on the growth rate in oil prices: it would have quantified the degree to which the current growth rate in real output depends on how far real output is 'out of kilter' with its long-run relationship with the price of oil, rather than quantifying the degree to which the current growth rate in real output depends on recent (but fairly smooth) fluctuations in the price of oil. In the event, however, such an error-correction term was not necessary, but it is important to understand the meaning which its coefficient would have had.[29]

In principle, one could instead model the relationship between real output and the oil price in levels directly, instead of using a level model only in order to find/estimate a potential

[29] As shown in Ashley and Patterson (2010) the fractional integration alternative – 'first-differencing' both real output and the oil price to a fractional exponent – removes any slow, smooth variation in the means of both time series prior to the analysis. This alternative is unattractive because – in common with any high-pass bandpass filter – it simply eliminates the low frequency variation in both time series. It also has the disadvantage of leaving the modeling in terms of variables (fractionally differenced time series) which are difficult to interpret economically.

cointegrating vector. Of course, one would first remove any linear trend from each time series, as regressing trended time series on one another is a well-known source of spurious regressions. That de-trending inherently makes both series at least appear to be mean-reverting over the sample period.[30] Estimates of the coefficients in the level model (e.g., for the cointegrating vector) are known to be consistent, which implies that the estimators converge in probability to population values for arbitrarily long samples. But our actual sample – while perhaps in the hundreds – is not at all 'arbitrarily large.'

Indeed, there are compelling intuitive reasons for thinking that even quite large samples are substantially smaller than one might think when the data are highly persistent. For example, let $x(t)$ denote the deviations of, say, T observations on some highly-persistent – albeit perhaps I(0) – time series from its trend. Because of its high persistence, a time plot of $x(t)$ loops around its (sample) mean value of zero only a handful of times over the course of the sample. Let j denote the number of such loops. Effectively, then, most of the sample information on $x(t)$ can be summarized by $2j$ numbers: the value of $x(t)$ on each maximal excursion and the time at which each such excursion occurs. But the value of j is likely to be far, far smaller than T. Thus, if both real output and the oil price are highly persistent, then our levels-model regression equations effectively are based on j observations rather than on T observations. They are still consistent, because j become arbitrarily large as T grows unboundedly. But one can expect poor results from such regressions in levels, even though T is quite large.[31]

[30] An I(1) time series is not actually mean-reverting, but its deviations from an estimated linear trend will necessarily appear to be so over the sample period used in estimating the trend. A highly persistent I(0) time series is in fact mean-reverting over all sample periods and will thus appear to be so over any sufficiently lengthy sample period, including the one used in estimating the trend.

[31] Ashley (2012, Chapter 14) takes up this topic in much greater detail. In particular, it examines the issues involved in obtaining meaningful standard error estimates for regression coefficients estimated using highly persistent data.